Bloch-Nordsieck violating electroweak corrections to inclusive TeV scale hard processes

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We point out that, since the colliders initial states (e^+e^-, pp, $p\bar{p}$, ...) carry a definite nonabelian flavor, electroweak radiative corrections to inclusive hard cross sections at the TeV scale are affected by peculiar Bloch-Nordsieck violating double logs. We recall the setup of soft cancellation theorems, and we analyze the magnitude of the noncancelling terms in the example of electron - positron annihilation into hadrons.

Interest in logarithmically enhanced electroweak corrections at NLC energies has recently arisen [1–5], after the observation - made by two of us [1] - that double and single [2] logarithms of "soft" Sudakov type are present and sizeable in fixed angle fermion-antifermion annihilation processes at the TeV scale. Such logarithms occur because at energies $\sqrt{s}$ much larger than the EW scale $M_W \approx M_Z \approx M$, the latter acts as a cutoff for the collinear and infrared (IR) divergences that would be present in the vanishing M limit.

The study of Sudakov form factors in the fixed angle, high energy, regime where the expansion parameter is $\alpha_W(\log \frac{s}{M^2})^2$ is actually a challenging problem, because one has to investigate the electroweak theory in a transition region between broken and unbroken theory, in which two mass parameters, the effective photon mass and the weak bosons mass $M_W \approx M_Z \approx M$ may both be important. Attempts to generalize by various approximation methods the one loop results to higher orders [3–5] have been made, but with somewhat controversial results[1]. Further study is then needed to fully clarify this point.

The fact remains, however, that the NLC regime is one in which the electroweak theory acquires, because of the enlarged phase space, a full-fledged non abelian structure, therefore giving rise to novel physical features with respect to the LEP regime. In this note we wish to point out the fact that even *inclusive* TeV scale cross sections are affected by Sudakov *double logs*, due to a lack of cancellation of virtual and real emission electroweak contributions.

In principle, this lack of cancellation is well known, and is due to the violation of the Bloch-Nordsieck (BN) theorem [6] in nonabelian theories. It was initially pointed out for QCD [7], where it would imply genuine IR divergences at partonic level, but was found eventually to have no physical consequences because of the color averaging of the initial partonic states, forced by the coupling to colorless hadrons. For instance, non factorized IR singular terms do occur in Drell-Yan processes , but at next-to-leading level only, where they are possibly suppressed by a Sudakov form factor [8], and eventually turn out to be higher twist [9,10].

On the other hand, in the electroweak case the initial state is *fixed* and carries a *non abelian charge*. Therefore, no averaging is possible, so that the BN violating terms - though finite because of the symmetry breaking scale M- are unavoidable for any inclusive observable and need to be carefully computed.

In order to understand this problem, let us start recalling the structure of soft interactions accompanying a hard SM process, of type

[1]For instance, Refs. [4] and [5] agree on the observation that photon and weak boson contributions have to be considered together to restore the gauge symmetry at very large energies, but differ in the evaluation of the symmetry breaking contributions.

$$\{\alpha_1^I p_1^I, \alpha_2^I p_2^I\} \to \{\alpha_1^F p_1^F, \alpha_2^F p_2^F, \dots, \alpha_n^F p_n^F\} \tag{1}$$

where α, p denote flavor/color and momentum indices of the initial and final states, that we collectively denote by $\{\alpha_I p_I\}$ and $\{\alpha_F p_F\}$. The S matrix for such a process can be written as an operator in the soft Hilbert space $\mathcal{H}_S$, that collects the states which are almost degenerate with the hard ones, in the form

$$S = \mathcal{U}^F_{\alpha_F \beta_F}(a_s, a_s^\dagger) \quad S^H_{\beta_F \beta_I}(p_F, p_I) \quad \mathcal{U}^I_{\alpha_I \beta_I}(a_s, a_s^\dagger) \tag{2}$$

where $\mathcal{U}^F$ and $\mathcal{U}^I$ are operator functionals of the soft emission operators $a_s, a_s^\dagger$.

Eq. (2) is supposed to be of general validity [11], because it rests essentially on the separation of long-time interactions (the initial and final ones described by the $\mathcal{U}$'s), and the short-time hard interaction, described by S^H. The real problem is to find the form of the $\mathcal{U}$'s , which is well known in QED [12], has been widely investigated in QCD [13], and is under debate in the electroweak case [4,5]. Their only general property is unitarity in the soft Hilbert space $\mathcal{H}_S$, i.e.

$$\mathcal{U}_{\alpha\beta}\mathcal{U}^\dagger_{\beta\alpha'} = \mathcal{U}^\dagger_{\alpha\beta}\mathcal{U}_{\beta\alpha'} = \delta_{\alpha\alpha'} \tag{3}$$

The key cancellation theorem satisfied by eq. (2) is due to Lee, Nauenberg and Kinoshita [14], and states that soft singularities cancel upon summation over initial and final soft states which are degenerate with the hard ones:

$$\sum_{i\in\Delta(p_I)}^{f\in\Delta(p_F)} |\langle f|S|i\rangle|^2 = \mathrm{Tr}_{\mathcal{H}_S}(\mathcal{U}^{I\dagger} S^{H\dagger} \mathcal{U}^{F\dagger} \mathcal{U}^F S^H \mathcal{U}^I) = \mathrm{Tr}_{\alpha_I}(S^{H\dagger}(p_F, p_I) S^H(p_F, p_I)) \tag{4}$$

where $\Delta(p_I, p_F)$ denote the sets of such soft states, and we have used the unitarity property (3).

Although general, the KLN theorem is hardly of direct use, because it involves the sum over the initial degenerate set, which is not available experimentally. In the QED case, however, there is only an abelian charge index, so that $\mathcal{U}_I$ commutes with $S^{H\dagger}S^H$, and cancels out by sum over the final degenerate set only. This is the BN theorem: observables which are inclusive over soft final states are infrared safe.

If the theory is non abelian, like QCD or the electroweak one under consideration, the BN theorem is generally violated, because the initial state interaction is not canceled, i.e., by working in color space,

$$\sum_{f\in\Delta(p_F)} |\langle f|S|i\rangle|^2 = \ {}_S\langle 0|\mathcal{U}^{I\dagger}_{\alpha_I\beta_I'}(S^{H\dagger}S^H)_{\beta_I'\beta_I}\mathcal{U}^I_{\beta_I\alpha_I}|0\rangle_S = (S^{H\dagger}S^H)_{\alpha_I\alpha_I} + \Delta\sigma_{\alpha_I} \tag{5}$$

where the α_I indices are not summed over and $\Delta\sigma_{\alpha_I}$ is, in general, nonvanishing and IR singular. Fortunately, in QCD the BN cancellation is essentially recovered because of two features: (i) the need of initial color averaging, because hadrons are colorless, and (ii) the commutativity of the leading order coherent state operators ($\mathcal{U}^l$) for any given color indices [13]:

$$\mathcal{U}^l = \mathcal{U}^l(a_s - a_s^\dagger) \quad , \quad [\mathcal{U}^l_{\alpha\beta}, \mathcal{U}^l_{\alpha'\beta'}] = 0 \tag{6}$$

We obtain therefore

$$\sum_{color} \mathcal{U}^{l\dagger}_{\alpha_I\beta_I'}(S^{H\dagger}S^H)_{\beta_I'\beta_I}\mathcal{U}^l_{\beta_I\alpha_I} = \sum_{color}(S^{H\dagger}S^H)_{\beta_I'\beta_I}\mathcal{U}^l_{\beta_I\alpha_I}\mathcal{U}^{l\dagger}_{\alpha_I\beta_I'} = Tr_{color} S^{H\dagger}S^H \tag{7}$$

thus recovering an infrared safe result (for subleading features, see Refs. [9,10,13]).

In the electroweak case, in which M provides the physical infrared cutoff, there is no way out, because the initial state is prepared with a fixed non abelian charge. Therefore eq. (5) applies, and double log corrections $\sim \alpha_W \log^2 \frac{s}{M^2}$ must affect any observable associated with a hard process, even the ones which are inclusive over final soft bosons. This fact is surprising, because one would have expected such observables to depend only on energy and on running couplings, while the double logs represent an explicit M (infrared cutoff) dependence not yet found before.

In order to compute the uncanceled double logs, a few preliminary remarks are in order. First, we assume the underlying process to be hard, involving a scale much larger than M. Therefore the lowest order soft contributions to $\Delta\sigma$ in eq. (5) can be simply described by the external (initial) line insertions of the eikonal current

$$J_a^\mu = \frac{p_1^\mu}{p_1 k} t_1^a + \frac{p_2^\mu}{p_2 k} t_2^a \tag{8}$$

as depicted in Fig. 1. Secondly, we start from the first non trivial order, where the effect is present and easily understandable. Then, the calculation is obviously gauge invariant, because in the hard (Born) cross section the symmetry is restored, and weak isospin is conserved. For instance, the weak isospin charge resulting from the squared insertion current (8) in the Feynman gauge and from Fig. 1 is

$$(\boldsymbol{t}_1 - \boldsymbol{t}_1') \cdot (\boldsymbol{t}_2 - \boldsymbol{t}_2') = -(\boldsymbol{t}_1 - \boldsymbol{t}_1')^2 = 2\boldsymbol{t}_1 \cdot \boldsymbol{t}_1' - \boldsymbol{t}_1^2 - \boldsymbol{t}_1'^2 \tag{9}$$

The last expression, obtained using isospin conservation, is identical to the axial gauge result. From this form of the charge it is clear that the Z_0 and γ contributions cancel out between real and virtual terms, and only the W contribution remains, which is coupled to left handed fermions only. By adding the obvious eikonal radiation factor, we finally obtain the following formulas for the corrections to the Born cross sections $\sigma_{e^+e^-}$ for the hard process defined by eq. (1):

$$\Delta\sigma^{RR}_{e^+e^-} = 0 \tag{10}$$

$$\Delta\sigma^{LL}_{e^+e^-} = -\Delta\sigma^{LL}_{e^-\bar{\nu}} = \mathcal{A}_W(s)\,(\sigma^{LL}_{e^+\nu} - \sigma^{LL}_{e^+e^-}), \qquad \mathcal{A}_W(s) = \frac{\alpha_W}{4\pi}\log^2\frac{s}{M^2} \tag{11}$$

where s is the c.m. energy squared, $\alpha_W = g^2/(4\pi)$, L, R refer to the initial fermions chiralities and where use has been made of the isospin conservation constraints $\sigma^{LL}_{e^+e^-} = \sigma^{LL}_{\nu\bar{\nu}}$, $\sigma_{e^+\nu} = \sigma_{e^-\bar{\nu}}$.

Eq.(11) provides a rather general result because the W coupling is universal and because only the initial state needs to be specified, so that it applies to various kinds of hard processes of the type of eq. (1). It is also clear that the double log cancellation is recovered by summing over flavors ($\Delta\sigma_{e^+e^-} + \Delta\sigma_{\nu e^+} = 0$). The actual magnitude of the effect is however dependent on the hard process, because the cross section difference between initial flavors appears in the r.h.s. of eq.(11). To be definite, consider the example of $e^+e^- \to q\bar{q} + X$, i.e. the total cross section associated to two jets with large transverse energy[2]. The Born amplitude in the hard symmetric limit is $\sim (g^2\boldsymbol{t}\cdot\boldsymbol{T} + g'^2 yY)/s$ where T (and Y) denote the quark weak isospin (hypercharge). By squaring and summing over final flavors, we obtain the cross section difference (N_f is the number of families[3] and N_c is the number of colors):

$$\sigma^{LL}_{e^+\nu} - \sigma^{LL}_{e^+e^-} = \frac{N_f N_c}{12\pi\, s}\left(\frac{g^4}{8} - \frac{g'^4}{4}Y^2\right) \qquad \left(Y^2 = 2\,Y_L^2 + Y_R^2 + Y_{R'}^2 = 2\frac{1}{36} + \frac{4}{9} + \frac{1}{9}\right) \tag{12}$$

which occurs in eq.(11), and yields

$$\frac{\Delta\sigma^{LL}_{e^+e^-}}{\sigma^{LL}_{e^+e^-}} \simeq 0.8\,\mathcal{A}_W(s) \tag{13}$$

In this case the non canceling terms are positive for initial e^+e^- beams and no particular suppression is noticed. Given the size of this effect, it is advisable to compute higher orders as well. We think that, due to the inclusive nature of the measurement, the QED scale cannot play, in this case, an important role. It is then tempting to compute higher orders on the basis of the leading form (6) of the coherent state operator [13] with gauge group $SU(2) \times U(1)$ and only one cutoff, that is M. With this assumption, a straightforward calculation shows that γ and Z contributions cancel out in the general case as well, and the W contribution exponentiates in eq.(11) in the form [15]

$$\mathcal{A}_W(s) \to \frac{1}{2}\,(1 - e^{-2\,\mathcal{A}_W(s)}) \tag{14}$$

This means that, when the energy increases, the noncancellation in eq. (14) becomes maximal and the initial state Sudakov effects equalize eventually the electron and neutrino beam cross sections. The universal exponent appearing in eq. (14) is that of the Sudakov form factor in the adjoint representation, whose relevance was noticed for QCD by Mueller [8] and by Catani et al [9], and is proved for the present case in [15].

So far, we have concentrated on lepton colliders, because EW double logs are more directly relevant in such a case. It is amusing to note that the non canceling terms affect hadron colliders as well, because hadrons carry EW charges

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[2] We are not concerned here with experimental subtleties, coming from contamination with four jet events coming from two boson production, or with the competing boson fusion mechanism, which starts at higher perturbative order.

[3] The special case of the top quark, requiring a heavy mass cutoff, was considered in [2], but leads to no important differences at the double log level we are working.

too. For instance, at parton level, a formula like (11) holds with an initial quark doublet also, while EW effects wash out in the quark-gluon and, of course, in the gluon-gluon cases. We think therefore that such EW double logs are to be seriously considered for LHC too.

To sum up, TeV scale accelerators open up a regime in which we really see non abelian charges at work: even inclusive cross sections have large electroweak corrections. For instance, in our example of e^+e^- into hadrons, while QCD corrections are $O(\frac{\alpha_s}{\pi})$, the electroweak ones are $O(\frac{\alpha_W}{4\pi}\log^2\frac{s}{M^2})$, which increases with energy and is already 7% at the TeV threshold.

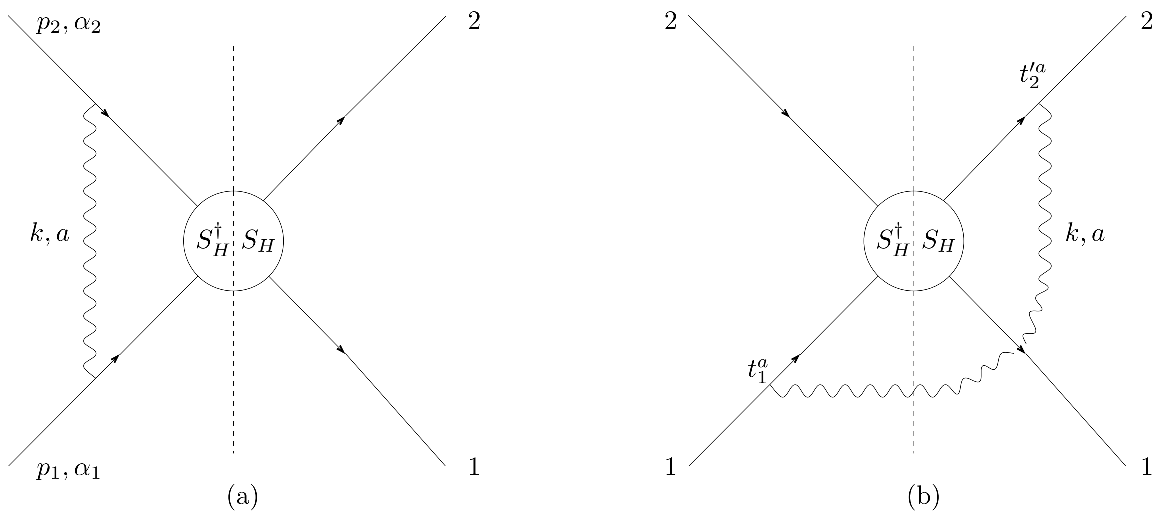

FIG. 1. Unitarity diagrams for (a) virtual and (b) real emission contributions to lowest order initial state interactions in the Feynman gauge. Sum over gauge bosons a= γ, Z, W and over permutations is understood.

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